LOGICAL AND NATURAL CONNECTIVES

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1. One of the points where logical and natural languages seem to diverge is in their respective connectives. The set of logical connectives, interdefinable with negation (as a so-called 'one-place connective'), such as conjunction (A), disjunction (v) and material conditional (z), seem only in part related with their natural language counterparts and, or, if...then. Moreover, this latter set of grammatical 'conjunctions' contains other members such as but, although, for, because, whereas adverbs such as so, yet, nevertheless should also be taken as natural connectives, both groups without corresponding connectives in classical logics. Connectives of classical systems, it should be recalled, are truth functional: given the truth values (truth or falsity) of component propositions (or sentences) they determine the over-all truth value of the compound proposition (or sentence). Axiomatically, they have properties of commutativity, associativity, distributivity (conjunction, disjunction) or they are transitive (conditionals).

2 The first problems arise as soon as connectives are used in non-truth-functional contexts, e.g. with modalities. The connectivity of two propositions a and 0 is not normally restricted by the 'content' of a or 0; only their truth values count, not the fact whether they are somehow semantically related in another way. As soon as we prefix the box (0 )—denoting 'necessity' (truth in all possible accessible worlds)—to the compound formula, e.g. as follows:0(uo $), which may also be written as a-30, where ' denotes a so-called 'strict' conditional or implication, the question arises in which respect propositions or their values can be said to be 'necessarily' connected. If we say, for example, that a necessarily or logically implies $, we might want to require that this makes sense only if W and 0 have some inherent relation, to be interpreted as a corresponding relation between truth values or facts. Since there is a systematic relation between conditional connectives and the relation of deductive inference (t-- ) by the deduction theorem (af-0 ml-a=0), similar problems arise in the conditions of appropriate deduction. If, for instance, a conclusion is logically implied by the premises, we may also desire to put some constraints on the nature of the premises, e.g. that they have at least a proposition in common with the conclusion. The conflict with the validity of formulas such as ED(a=2), $=(2Da), (EA -(2v-2)), (2D (E v a) and ((2vg)A-p)/7.5, then becomes apparent, because they enable
to derive statements about the truth value of \( \mathcal{E} \) or \( \mathcal{g} \) (or of a relation between them), without knowing the truth value of one of them. These and other intuitive arguments have led to call these theorems the paradoxa of material implication, which seem still more paradoxical in their strict implicational versions.

In order to avoid these and related counter-intuitive properties of certain logical connectives (and deduction), several types of so-called relevance logics (or connective/connexive logics) have been developed, with which are associated the flanes of, among others, Ackermann, Anderson, Dunn, Belnap, Church, Gabbay, (David) Lewis, McCall, Meyer, Routley, Stainaker, Urquhart. The main idea behind these, often rather diverse, approaches is to design axiomatic systems and/or formal semantics for relevant connectives, especially relevant conditionals ('normal' and strict).

Intuitively, then, the antecedent of such a conditional must 'have something to do' with the consequent, for example due to a meaning relation. Similarly, of course, for deductive inference, i.e. between premises and conclusions.

Thus, we may require that a formula with a relevant conditional, \( \mathcal{E} > \mathcal{a} \), corresponding to our natural language intuitions for a conditional such as if...then, would take truth values only if \( \mathcal{E} \) is true (and/or "asserted").

3. The debate about logical connectives and their required relevance especially focuses on the properties of entailment, taken as a (semantic) relation between propositions. If \( \mathcal{a} \models \mathcal{b} \), then if \( \mathcal{a} \) is true, \( \mathcal{b} \) is also (necessarily) true. In such a case, it is assumed, a necessary semantic dependency of the propositions must involve conceptual relations between the propositions. If this is so, we can hardly take the normal strict implication (\( \rightarrow \)) or deductive inference (\( \vdash \)), as the formal syntactic counterparts of entailment, because there, classically, no conceptual (intensional) constraints exist on connectivity or deduction. Hence, it is argued, we should take relevant (strict) conditionals as the syntactic expression for semantic entailment; especially if we would like to say that, in inferences, conclusions are 'entailed' by their premises, and not only 'follow logically from' their premises. Since we here talk 'about' conditions on deduction, the notion of relevance, as expressed in specific connectives, is also often relegated to the logical meta-language. Much of the current work on (relevant) connectives is devoted to the construction of various entailment logics.
A crucial issue, of course, is to develop a serious formal semantics for the various relevant or intensional connectives. The basic idea behind the different semantic accounts is that the models contain elements which enable interpretation of propositions (sentences) relative to each other—instead of freely assigning them independent values as in classical systems. Thus, we may require that the respective possible worlds in which the propositions are interpreted are somehow 'connected', e.g. with respect to a third possible world, a (communicative) context, or a set of propositions (information).

Similarly, if a proposition \( \phi \) should be interpreted relative to (the interpretation of) \( \alpha \), we may let some function, determined by \( \alpha \), select the possible worlds where \( \phi \) may have truth values. One of the many open problems, both axiomatical and semantic, is a unified treatment of the various relevant or in general intensional connectives, including conjunction and disjunction, and a fundamental semantic device for 'relative interpretations'. Perhaps recent developments in intensional logic may lead to a solution for these problems.

4. Since, in natural language and theories about it, we are not only interested in truth conditions or other extensions, but also in the meaning structure of propositions and in their intensional inter-relations, natural connectives as those mentioned above, essentially seem to require a formal reconstruction in terms of 'relevant' connectives. This holds for all connectives, including conjunction and disjunction, even if, occasionally and under specific constraints, we may connect two propositions which are neither conceptually nor referentially related. More in general, it should be required that conditions of connection are satisfied by each compound sentence (and connected sequences of sentences in a discourse, for that matter). Although meaning relations may be involved in the coherence of compound sentences and discourse, the connection conditions rather depend on the relations between the values of the propositions in some possible world(s), i.e. on the relations between the 'facts'. These facts, in general, should actualize the same conceptual range in semantic space—i.e. the fact-concepts, or propositions, must belong to the same range. Relative to a given possible world, point of view or topic of discourse the facts must be compatible in related (similar, identical, successive) worlds. Whereas the weakest form, then, of natural conditionals satisfying these conditions is 'possibility' (e.g. in co-occurrence or 'allow'-condition) as expressed by conjunctions, the stronger connection is that of pro-
bability (likelihood) as in concessives such as but (where a
but $S$, implies if $a$ then normally [in most possible worlds]—
$:lr$ and finally necessity as in causals and implications such as
because, for, so, therefore etc., where the antecedent
logically, physically, biologically, psychologically or
socially 'necessitates' the consequent in a certain possible
world (or in most, or all possible worlds). Necessitation,
in such a case, holds, we will say tentatively, in a world
$w$, where $a$ is true, if $b$ is true in all successive worlds $w^*$
which can be reached from $w$ (immediately, or sometimes
indirectly). If Peter falls from his chair, this may cause
that he breaks his neck, necessarily in that course of events
(given the precise physical, and other circumstances), but of
course not in all possible worlds where Peter falls from his
chair, as would physically be the case for the connection
between 'rising temperature (above value $x$)' and 'melting of
butter'.
Among these referential and intensional conditions on the
connectivity of propositions, we have briefly mentioned that
of 'information' or 'topic'. This means that not only the
facts or the meanings must be related, but this must be so
relative to what is known (by the speech participants) in a
certain context and by the topic of discourse at issue, i.e.
one or more propositions defining the 'over-all' meaning of
compound sentences and discourse (macro-structure). Besides
the conceptual constraints on the topicality of propositions,
we have other conventional constraints related to the knowl-
dge of the participants about 'typical structures' of the
world (e.g. that we order food in restaurants, take an
umbrella when it rains, etc.--dependent on culture). Such
conventional knowledge structures are called frames.
In a formal semantic of natural connectives we should build
in these specific conditions and constraints on natural
connection. But at the moment little insight exists in the
possible forms such an extension of formal semantics should
take, let alone the account of the pragmatic connection con-
ditions.
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